

**Workshop on Compressible Navier-Stokes Systems and Related Problems (I)**

**March 5-10, 2018**

**TITLE & ABSTRACT**

*(Last updated: 6 March 2018)*

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**Classification of asymptotic states for radially symmetric solutions of multi-dimensional Burgers equation**

**Itsuko HASHIMOTO**

*Osaka City university & Kansai University*

We consider the asymptotic behavior of radially symmetric solutions for the multi-dimensional Burgers equation on the exterior domain in multi-dimensional space, where the boundary and far field conditions are prescribed. We show that even for some case where the corresponding 1-D Riemann problem for the non-viscous part admits a shock wave, the solution tends toward a linear superposition of stationary and rarefaction waves as time goes to infinity. Furthermore, we clarify the asymptotic stability of the stationary waves. Finally, for the case of  $n=3$ , we give the complete classification of the asymptotic behaviors, which includes even a linear superposition of stationary and viscous shock waves. This is the joint work with Prof. Akitaka Matsumura of Osaka university.

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**Concentration and oscillation phenomena in fluid dynamics**

**Xianpeng HU**

*City University of Hong Kong*

The weak stability is an important issue in fluid dynamics. We will discuss the recent mathematical understudying of concentration and oscillation phenomena in the framework of weak solutions. Two typical examples, including compressible Navier-Stokes equations and the incompressible viscoelasticity, will be considered.

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**Harmonic vector fields in  $L^r$  on 3D exterior domains**

**Hideo KOZONO**

*Waseda University*

In this talk, we characterize the space of harmonic vector fields in  $L^r$  on the 3D exterior domain with smooth boundary. There are two kinds of boundary conditions. One is such a condition as the vector fields are tangential to the boundary, and another is such one as those are perpendicular to the boundary. In bounded domains both harmonic vector spaces are of finite dimensions and characterized in terms of topologically invariant quantities which we call the first and the second Betti numbers. These properties are closely related to characterization the null spaces of solutions to the elliptic boundary value problems associated with the operators  $\text{div}$  and  $\text{rot}$ . We shall show that, in spite of lack of compactness, spaces of harmonic vector fields in  $L^r$  on the 3D exterior domain are of finite dimensions and characterized similarly to those in bounded domains. It will be also clarified a significant difference between interior and exterior domains in accordance with the integral exponent  $1 < r < \infty$ . This is based on the joint work with Profs. Hieber, Seyferd, Shimizu and Yanagisawa.

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**Small Debye length limit for the Euler-Poisson system**

**Bong Suk KWON**

*Ulsan National Institute of Science and Technology*

We discuss existence, time-asymptotic behavior, and quasi-neutral limit for the Euler-Poisson equations. Specifically we construct the global-in-time solution near the plasma sheath, and investigate the properties of the solution. If time permits, some key features of the proof and related problems will be discussed. This is joint work with C.-Y. Jung (UNIST) and M. Suzuki (Nagoya Inst. Tech.).

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**Behaviors of Navier-Stokes(Euler)-Fokker-Planck equations**

**Hailiang LI**

*Capital Normal University*

We consider the behaviors of global solutions to the initial value problems for the multi-dimensional compressible Navier-Stokes(Euler)-Fokker-Planck equations. It is shown that due the micro-macro coupling effects, the sound wave type propagation of this NSFP or EFP system for two-phase fluids is observed with the wave speed determined by the two-phase fluids. This phenomena can no be obsered for the pure Fokker-Planck equation.

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**Global Existence of Weak Solutions to the Barotropic Compressible Navier-Stokes Flows with Degenerate Viscosities**

**Jing LI**

*Chinese Academy of Sciences*

We consider the existence of global weak solutions to the barotropic compressible Navier-Stokes equations with degenerate viscosity coefficients. We construct suitable approximate system which has smooth solutions satisfying the energy inequality, the BD entropy one, and the Mellet-Vasseur type estimate. Then, after adapting the compactness results due to Bresch-Desjardins (2002, 2003) and Mellet-Vasseur (2007), we obtain the global existence of weak solutions to the barotropic compressible Navier-Stokes equations with degenerate viscosity coefficients in two or three dimensional periodic domains or whole space for large initial data. This, in particular, solved an open problem proposed by Lions (1998). This is a joint work with Prof. Zhouping Xin (CUHK).

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**Entropy-bounded solutions of the compressible Navier-Stokes equations with vacuum**

**Jinkai LI**

*The Chinese University of Hong Kong*

The entropy is one of the fundamental states of a fluid and, in the viscous case, the equation that it satisfies is both degenerate and singular in the region close to the vacuum. In spite of its importance in the gas dynamics, the mathematical analyses on the behavior of the entropy near the vacuum region, were rarely carried out; in particular, in the presence of vacuum, either at the far field or on the physical boundaries, it was unknown if the entropy remains its boundedness. It will be shown in this talk that the ideal gases retain their uniform boundedness of the entropy, locally or globally in time, for both the Cauchy problem and the initial-boundary value problems, if the vacuum occurs only at the far field or on the physical boundary, as long as the initial density behaves well at the far field or near the boundary. For the Cauchy problem, the density is required to decay slowly enough at the far field, while for the initial-boundary value problem, the  $(\gamma - 1)$ -th power of density is required to be equivalent to the distance to the boundary, near the boundary.

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**Global existence and exponential stability for the compressible Navier-Stokes equations with discontinuous data**

**Chuangchuang LIANG**

*Chongqing University*

The initial boundary value problem for the compressible barotropic Navier-Stokes equations is investigated in the case that the initial density has a jump discontinuity across an interior closed curve in two-dimensional bounded domain. If the initial data is a small perturbation of the constant state and the interior closed curve is near a circle inside the domain, the global existence and large time behavior of the piecewise strong solution is shown, in particular, the jump of the fluid density across the convecting curve decays exponentially in time.

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**Problem in Quantum Hydrodynamics**

**Pierangelo MARCATI**

*Gran Sasso Science Institute*

I will present an outline of results in quantum hydrodynamics, regarding motivations , problems and applications.

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**Asymptotic stability of a rarefaction wave for a symmetric hyperbolic-parabolic system arising in compressible fluid dynamics**

**Shinya NISHIBATA**  
*Tokyo Institute of Technology*

In this talk, we discuss a large time behavior of a solution to a coupled system of viscous and inviscid conservation laws. This system of equations appears in compressible fluid dynamics. We, mainly, talk about an asymptotic stability of a rarefaction wave under assuming the existence of an entropy function. This assumption enables us to transform the original system to a normal form of symmetric hyperbolic-parabolic systems. In asymptotic analysis, we derive an a priori estimate by an energy method. In order to derive the basic estimate, we make use of an energy form, which is obtained by substituting a smooth approximation of the rarefaction wave in the entropy function. The symmetric system is utilized in deriving the estimates of the higher order derivatives of the solution. In this procedure, we also have to suppose that the stability condition hold at spatial far field.

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**Singular limits of compressible Navier-Stokes equations**

**Yaobin OU**  
*Renmin University of China*

In this talk, I will present some results on the singular limits of compressible Navier-Stokes equations. For the full compressible Navier-Stokes equations in a three dimensional bounded domain, we establish the uniform estimates with respect to the Mach number in  $(0, 1]$  and the time in  $(0, +\infty)$  for the global strong solutions. Then as the Mach number tends to zero, the solution will converge to the one for isentropic incompressible Navier-Stokes equations. I also present a recent result on the low Mach number and Froude number limit for the isentropic Navier-Stokes equations with moving boundary. Both the cases of ill-prepared initial data and well-prepared initial data are discussed.

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**Quasiconvex elastodynamics: weak-strong uniqueness for measure-valued solutions**

**Stefano SPIRITO**  
*University of L'Aquila*

This talk concerns a classical system of conservation laws arising in elasticity. One of the main difficulty in elastodynamics is that convex stored energy functions are in contradiction with the basic physical principle of frame-indifference. In this talk I will present a recent result, obtained in collaboration with Kostantinos Koumatos University of Sussex (UK), proving the weak-strong uniqueness of measure-valued solutions under the assumption that the stored-energy function is strongly quasiconvex. The proof combines the relative entropy method and several tools borrowed from the calculus of variations.

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**Subsonic and Sonic Jet Flows for Given Surrounding Pressures from Convergent Nozzles**

**Chunpeng WANG**

*Jilin University & The Chinese University of Hong Kong*

This talk concerns the compressible subsonic and sonic jet flows for a given surrounding pressure from a two-dimensional finitely long convergent nozzle with straight solid wall. For a given surrounding pressure and a given incoming mass flux, we seek a subsonic or sonic jet flow with the given incoming mass flux such that the flow velocity at the inlet is along the normal direction, the flow satisfies the slip condition at the wall, and the pressure of the flow at the free boundary coincides with the given surrounding pressure. The well-posedness is shown and the properties of the flow are investigated.

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**Asymptotic limits of drift diffusion equations for semiconductors and the related models**

**Shu WANG**

*Beijing University of Technology*

I will discuss the asymptotic limit problems on some models arising from applied sciences such as semiconductor and plasma. The models include drift diffusion models and PNP models etc, which usually concern small parameter regimes. Some new results on rigorous limits like quasi neutral limit are given in this talk.

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**Global well-posedness of classical solution to the two-dimensional compressible Navier-Stokes equations**

**Yi WANG**

*Chinese Academy of Sciences*

First, we are concerned with the global well-posedness of the classical solution to the 2D Cauchy problem of the compressible NavierStokes equations with arbitrarily large initial data and non-vacuum far-fields when the shear viscosity  $\mu$  is a positive constant and the bulk viscosity  $\lambda = \rho^\beta$  with  $\beta > \frac{4}{3}$ . The initial data can be arbitrarily large with or without vacuum states. For the non-vacuum initial data, our global well-posedness result implies that the classical solution to the 2D Cauchy problem will not develop the vacuum states in any finite time. Furthermore, the global well-posedness result still holds true when the initial data contains the vacuum states in a subset of  $\mathbb{R}^2$  provided some compatibility conditions are satisfied. Some new weighted estimates for the density and the velocity are obtained to prove our main result and these self-contained estimates reflect the fact that the weighted density and velocity can propagate along with the flow, which are intrinsic to the two-dimensional Cauchy problem with the non-vacuum far-fields. Then I will talk about our recent result on the time-asymptotic stability of planar rarefaction wave to two-dimensional compressible Navier-Stokes equations in the domain  $\mathbb{R} \times \mathbb{T}$ , which gives a first result about the stability the planar rarefaction wave for the multi-dimensional viscous fluids with physical constraints.

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**Local well-posedness of the free boundary problem of the full compressible Navier-Stokes equations in 3D**

**Yuan YUAN**

*The Chinese University of Hong Kong*

In this talk we will present the local-in-time well-posedness of strong solutions to the free boundary problem of the full compressible Navier-Stokes equations in three-dimensional space. The vanishing density and temperature condition is imposed on the free boundary, which captures the motions of the non-isentropic viscous gas surrounded by vacuum with bounded entropy. Here we assume some proper decay rates of the density towards the boundary and singularities of derivatives of the temperature across the boundary on the initial data, which coincides with the physical vacuum condition for the isentropic flows. This extends the previous result of Liu [ArXiv:1612.07936] by removing the spherically symmetric assumption and considering more general initial density and temperature profiles. This is a joint work with LIU Xin from Texas A&M.

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**Nonlinear Asymptotic Stability of the Lane-Emden Solutions for Navier-Stokes-Poisson Equations**

**Huihui ZENG**

*Yau Mathematical Sciences Center*

In this talk, I will present some results joint with Tao Luo and Zhouping Xin on the nonlinear asymptotic stability of the Lane-Emden solutions for spherically symmetric motions of viscous gaseous stars in the setting of vacuum free boundaries capturing the physical behavior that the sound speed is  $1/2$ -Holder continuous across the vacuum boundary. The key issue is to establish the global-in-time regularity uniformly up to the vacuum boundary due to the difficulty caused by the degeneracy and singular behavior near the vacuum states. The main ingredients of the analysis consist of combinations of some new weighted nonlinear functionals and space-time weighted energy estimates. The constructions of these weighted nonlinear functionals and space-time weights depend crucially on the structures of the Lane-Emden solution, the balance of pressure and gravitation, and the dissipation.